Mathematics, Grade 8 Unit XI: Lesson A1

Model: Chords of Equal Time

6.15%

Galileo, in studying motion, theorized that the times of descent of objects along radial chords of a vertical circle are equal. A vertical circle is a circle that is standing upright. θ is the Greek symbol Theta, which is commonly used as a variable for angles.

Construct a working model that will demonstrate this theorem. On a piece of wood, Masonite or other sturdy material, scribe a circle. Insert nails or screws and stretch wires between them to represent the chords. Place a small bead on each wire. If the beads are released from the top of the circle simultaneously, they will hit the circumference at the same time. Turn the circle over and release several beads at the same time. They will arrive at the common point simultaneously.

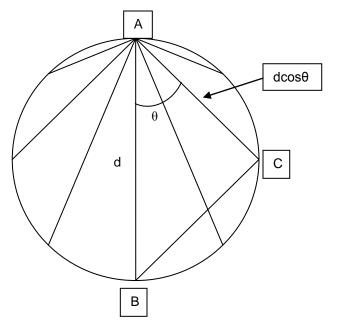
The reason for the simultaneous arrival is as follows:

Let AB = the diameter of the circle and AC any other chord. The angle ACB is a right angle.

The length of any cord is d (diameter) times $\cos \Theta$, Θ being a variable for an angle where the cord and the diameter meet.

The force of gravity is acting on the beads in a direction they can't go (straight down) because the wire is in the way, so the force is determined in two parts.

One part, CB, acts perpendicular to the wire. This is the direction the bead would swing if it were free to do so.



The other direction of force, AB, is parallel to the wire, tending to pull the bead down the wire. If AB is the force pulling the bead down the wire, then AB ($\cos \Theta$) is the force pulling the bead down the chord. The forces pulling the beads down the wires are proportional to the distances the beads travel on the wires.

Therefore, the time it takes the bead to descend one chord would be equal to any other chord of that same circle.

Use your model to verify this explanation. Write equations using your measurements. Show that the ratios form a proportion.

Demonstrate your model to the class using your data to explain what is happening. Account for variations in any beads that do not arrive simultaneously.